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# Discovery and Invention: The Newtonian Revolution in Systems Technology

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## Revolutions?

THIS year we are celebrating the dawn of flight: powered, controlled, manned flight. This was a revolution, but a very slow revolution. It began many millennia ago, with Daedalus, the first great engineer in the history of engineering.[1] And it was not the only technological revolution. Besides wanting to fly like a bird, man also fancied swimming like a fish (underwater), rising to the stars (as in the imagination of poets), and seeing/hearing the past (movies/television/sound recording). About 150 years ago, things started to liven up, and we have had three or four simultaneous and interrelated technological revolutions. They were foreseen and romanticized by Jules Verne (1828–1905), the father of technology-fiction, favorite writer of my youth, and also that of my engineer father. Verne's visions came to be the prehistory of what followed, as in *Robur, the Conqueror* (1886) (flying machine/helicopter), *Twenty Thousand Leagues under the Seas* (1870) (private submarine), *From the Earth to the Moon* (1865) and *Around the Moon* (1870) (space tourism), and *Castle in the Carpathians* (1892) (recording and recreation of opera in stereo sound and sight).

Like (probably) all contributors to our centennial celebration, I am in a poor position to say anything personal about the dawn of flight, having been born exactly half a century too late. [My S.B. degree, Massachusetts Institute of Technology (MIT), was in 1953.] Much the same holds for the other developments just mentioned. My admiration for the courage and initiative of the pioneers of flight is, so to say, boundless, but it is purely intellectual, not emotional. I wasn't present at the creation.

If I am to make an emotional/intellectual contribution to this celebration, I should look farther afield and bring back into memory a different revolution. The Newtonian revolution. A revolution that, in my days, has profoundly affected all of the other revolutions mentioned before; and much more: a revolution that is likely to be the most permanent of any of them. This is a revolution I have always liked. I have contributed to it.

It is not a revolution that Verne had anticipated. I would exclude here *The 500 Millions of the Begum* (1879) (in which a private weapon of mass destruction turns out to be useless because of inadequate mathematical modeling in the planning stage) as well as *Topsy–Turvy* (1889) (in which an attempt to reposition the axis of the Earth for private profit fizzles because of mathematical error). These stories did involve mathematics but had a very negative message. My message is totally positive.

All revolutions are not "created equal." One may shudder at the memory of the violence of the French Revolution (1789) not to speak of the brutality of the Russian Revolution (1917), but one should not overlook the bloodless Japanese Revolution (Meiji Restoration, 1867) which was far more important to the world and is still a major political force today. The Newtonian revolution seems to me to be similar.

[As regards the connection technology/revolution, let me cite here the admirable work of Lucio Russo.[2] He recounts the rapid expansion of technology and the sciences in the Hellenistic era, two millennia ago, a revolution, which to him is the "forgotten one." His thesis is that this revolution, by the year zero, ran into so many of economic, educational, political, social, moral, and intellectual (but mainly political) obstacles that it had to prematurely die. This scenario makes both of us terribly nervous because it brings to mind the situation we find in the year 2003.]

My exposition will emphasize intellectual history.

## What Newton Did

To the modern media consumer, Isaac Newton (1643–1727) appears as a disagreeable person. For example, reminiscent of Bill Gates, he thought he should be paid for intellectual work, even if of no immediate practical value. Never mind, he became very famous, even among the little educated. Because he did something absolutely unexpected, unimaginable, ununderstandable—but true. He made it believable that he (we) can unravel the secrets of the real world we live in by using nothing more than... Mathematics!

Nobody understood this, of course, except, probably, the great man himself. (We shall return to this point later.) And they all believed it!

The poet Alexander Pope (1688–1744) wrote (see Ref. 3), in 1735,

> "Nature and Nature's laws lay hid in night;
> God said: Let Newton be! and all was light.

This is the usual story we read in textbooks written for educating physicists and engineers. And it does Newton a great and well-deserved honor.

But it misses the big ideas. What are called "Newton's laws of motion" in the textbooks were actually discovered in the course of the researches of Galileo (1564–1642), who died about nine months before Newton was, prematurely, born. Newton never claimed credit for what Galileo did. Newton's laws of motion are in essence a repackaging, in precise mathematical language, of Galileo's discovery of inertia. This job was preprogrammed by Galileo himself. Everyone knows that he kept on repeating, "the Laws of Nature are a book written in mathematical characters."

Newton wrote his magnum opus (in Latin, completed 1687), *Philosophiae Naturalis Principia Mathematica*, entirely within the Galilean/mathematical framework (note the last word in the title), but his immediate inspiration was different. As is well known, around 1682–1684, Newton started a line of "experiments," with pencil and paper, using nothing more than Euclidean geometry (Greek mathematics), all of which he knew exceedingly well, and without any help from new physical measurements, special apparatus, or a government-financed laboratory. His sole aim in this research was to understand the implications of the work of Kepler (1571–1630), those famous three laws of Kepler (1609, 1619), which first gave an accurate behavioral description of planetary motion. The results of this purely mathematical investigation were recorded in Newton's manuscript, still extant, "De Motu" (see Ref. 4).

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These experiments led Newton to the amazing discovery, no less amazing now than it had seemed at the time, that Keplerian motion, the motion of a planet around a sun conforming to Kepler's laws, was a direct, deductive, rigorous, and unique consequence of a centrally acting force inversely proportional to the square of the distance (inverse-square law). Everybody would of course identify this force with the very real and very mysterious force of gravity. Conversely, postulating the universal applicability of all three of Kepler's laws to the planet/sun problem, Newton proved, by mathematics only, that the force responsible for the motion had to be, necessarily, a central, inverse-square-law force.

Thus, gravitation, as a phenomenon well known already to Daedalus, was, after many millennia, at last seen to be what it really was, a force obeying a simple mathematical formula: inversely proportional to the distance squared. All of this was a result of Kepler's insight into behavioral astronomy combined with Newton's mathematics. And there knowledge about gravitation has rested, more or less, now for over 300 years.

It really boggles the mind. Here is a plain fact of physics every living child, dog, bird, etc. knows, and yet it takes gazing at the sky and playing with Greek mathematics before it can be understood. Miraculous: mathematics ⇒ physics?!

[So miraculous, there was no lack of serious people who refused to believe it, for example, in our days, the physicist Richard Feynman (1918–1988). Acting on his own robust prejudices, that "the role of mathematics in physics is, at best, trivial," he toyed with bypassing Newton's "arcane mathematics" as ununderstandable for a physicist and tried to get at Kepler's first law by "elementary" means, as though this was the burning issue of the 1680s or the 1960s. In a lecture to undergraduates (1964), Feynman barely managed to "motivate," let alone prove, Kepler's first law. (After centuries of mathematical progress, there are of course elementary proofs now, but not along the lines attempted by Feynman.) Blinded by his own prejudices and overkilling with "elementary," Feynman completely missed Newton's deep ideas because these were mathematical and not elementary, and certainly not "physical" in the 20th-century sense.

Feynman's prejudices were not without admirers. After he died, his friends reconstructed this particular lecture, from notes and photographs, and then published it as Feynman's "lost lecture."[5] (It was certainly not his last lecture.) This is how we were given a fine illustration that science does not always move forward; it is possible, and not too difficult, to make it move backward.]

[On the lighter side, Newton himself gave us his famous joke, the silly story of the apple: "On a moonlit night, watching an apple fall from a tree, after some seconds of thought, the main features of the gravitational law just popped into my mind"—a tall tale about a job that took him many years of concentrated effort. It seems he invented and circulated this story at the end of his life. To me this is a story sprung from a really nasty sense of humor of a man who was pessimistic about being able to explain how mathematics could instruct us about physics.]

## Deeper Implications

There is no question that the *Principia* did start things moving, in the 18th, 19th, and early 20th centuries. It was the main intellectual source of what is today's mathematical physics. But there was much more to the *Principia.*

Modern engineering, including aero/astronautics, has happily adopted the Newtonian recipe for success:

1) Collect or recollect your knowledge of physics.

2) Transcribe that knowledge into mathematical form—if you can. (This is called modeling.)

3) Experiment using mathematics, not physics. (This is called simulation.)

4) Dress up your conclusions as mathematics. (This is the reason for reports, computer programs, etc.)

The recipe is guaranteed to work. And you don't owe any thanks, let alone royalties, for it. There are innumerable worthy projects of simulation, modeling, model-based control, etc., etc. This may not be a true revolution, but it is undoubtedly a major growth industry.

After 300 years, we should be able to see that such things were no more than beneficial fallout from Newton's great discovery. The essence was elsewhere. To put this in sharp focus, let us rephrase Newton's discovery abstractly as a theorem:

> Kepler's laws ⇔ Newton's gravitational law

⇔ means isomorphic, mathematically the same thing.

On the left-hand side, there is an idealized, exact description of the Keplerian motion of a planet (in an elliptical orbit, with a very large sun immobile at one of the foci of the ellipse, etc.), a description deduced by Kepler from direct astronomical observations. On the right-hand side, there is a formula claiming to be the exact quantitative description of an invisible force, one whose direct measurement is conceivable only under special conditions. And the theorem says these two very different things are—abstractly—the same! Incredible! An even greater miracle than that celebrated by Pope!

Implicit in the presentation of the theorem (because the implications are bidirectional) is the claim that there is only one gravitational law and that, therefore, there is only one kind of motion (elliptical) of a (small) planet around a (big) sun. In other words, what is being claimed is that

> there is a *unique* connection between data concerning physical reality and physical reality itself.

Going even beyond this, Newton's theorem can be given still another phrasing, as follows: If you have good data (behavioral or experimental), there is exactly one model that will explain the data. This model behaves exactly like the given data, and the data can be reproduced by one and only one model. To find out what unique model goes with your data, you stop doing physics, you do mathematics. No more experimentation, just brainwork. (Of course brainwork, work for a brain, is to be interpreted nowadays as a job for computers.)

Now this is a great discovery! A really *revolutionary* discovery!

It seems that Newton never doubted that here was his finest intellectual achievement: that he could prove, mathematically, rigorously, deduce, that the famous Keplerian elliptic orbits of the planets all came from the Newtonian gravitational law. (As a bonus, it turned out that the hyperbolic orbits of the comets also came from the gravitational law.)

How to explain these miracles to others was an altogether more difficult problem. We do not, we cannot know how far Newton's thinking evolved in the direction of the modern conceptual framework we have outlined. We can infer, nevertheless, that he knew it was a great discovery. We know this is so because of his unsuccessful attempts to explain it. Some of these attempts are contained in the several drafts he wrote for a postscript intended for the second edition (1713) of the *Principia.* This was the famous Scholium Generale. Eventually, and I think, in desperation knowing he would be unable to say it exactly as he wanted to say it, he tossed out (but, fortunately, did not tear up) previous attempts at explanation and contented himself with the following cryptic half sentence of just three words, and these three words would soon become perhaps the most famous utterance in the history of Western science:

> "& hypotheses non fingo."

(This is all that appeared in print about the matter in the published version of the Scholium Generale.) Newton was then already over 70 years old.

Conventional wisdom reigning in the 20th century would have put it in exactly the opposite way: "A theoretician is someone willing to assume anything except responsibility." Newton, it would appear, wanted to make it very clear that he was not that kind of a theoretician.

For us, modern readers, his technical advice should be obvious:

> Build your models from data, not from assumptions.

[This was in fact less than obvious at first. The word "fingo" being crude Latin, it was politely mistranslated into English, just two years after Newton's death, as "I do not 'frame' hypotheses," which is plain nonsense. There is a big literature on this issue. Let us simply note here that the numerous earlier manuscript drafts of the Scholium Generale mentioned earlier (see Ref. 6) leave no doubt about what Newton had on his mind. It was something like this: I do not condescend to use faked assumptions taken from thin air; my results follow from established facts; anything else would be a prejudice and prejudice will never lead to good science.]

## Newton's Invention

There is one more aspect, enormously important, of Newton's theorem. It is not just a great discovery, it is, at the same time, a great invention. I shall try to explain.

The correspondence between data and model being something of a purely mathematical nature, the job of explicitly finding the model (equations and parameters) is also, necessarily, a job of a purely mathematical nature. And the solution of any mathematical problem—once someone, a professional mathematician or perhaps an intelligent computer, has worked out in detail all of the relevant principles—is a job that is automatable.

This is Newton's great invention:

> If you mathematize your data, mathematics (and the mathematician and the computer) will automate your model building and then all that which follows from it is also automatable.

This reads just like the description of a patent, but it is not altogether clear if Newton could have succeeded in getting a patent according to current patent law.

## Algebraization

The idea that models must be built from data and not from prejudiced assumptions is extremely tempting but easy to resist; unless you have expert help, like Newton had from Kepler, it is very hard to put data into an exact mathematical form. (Kepler's laws, which were Newton's data, have been, from the start, mathematical and exact.) On top of that, to automate model building, you must then also get yourself a catalog of models, all of them again in exact mathematical form. (Newton's gravitational law happened to be a very simple model, consisting of just one formula.)

Since Newton's time much of mathematics has become rather abstract, sometimes so abstract that its usefulness for real-world problems becomes questionable. Today, if we want to make practical use of Newton's pending patent, the first step, the mathematization of data, needs to be reinterpreted in a more restrictive sense. What we want is "algebraization." Algebra tends to be concrete. Calculus, like differential equations, tends to be less concrete.

No one has done more to algebraize physical dynamics, that is, differential equations, especially in the those realms of the physical world colonized by electrical engineers, than Oliver Heaviside (1850–1925). He was the first to convince electrical engineers that electric circuits have certain physical attributes—soon to be called *impedance*—that can be quantified and can be represented as indeterminates or complex numbers in ordinary algebraic computations, while at the same time measured like voltage and current.

[Heaviside called this his "operational calculus." It turned out to be an unfortunate and confusing name. The wrong word, calculus, misled some mathematicians of mediocre imagination to "dealgebraize" Heaviside's terrific algebraic idea and even, unkindly, change the name to Laplace transform. The new name had to do with reinventing a combinatorial-algebraic tool, known earlier as generating functions, developed a century before by Pierre-Simon Laplace (1749–1827). Those who worked on the Laplace transform managed to ruin, to dealgebraize, generating functions as well, but fortunately in the latter case the name of the inventor remained attached. Heaviside, largely autodidact, was a practical hands-on guy; he didn't have the excellent education and scientific culture acquired by Newton at the great university of Cambridge.]

Heaviside's contribution, that differential equations governing electrical circuits can be algebraized and called impedance, was undoubtedly a major achievement. Calculus, as a prestige word, will probably forever remain attached to Newton. But let's be clear: He didn't use calculus (differential equations) in his researches on Kepler's laws; in the *Principia*, he relied on a kind of global analysis.[7] This was of course forgotten by 1900, if it was ever clearly understood during the two preceding centuries. Thus, it is a fact that Heaviside's independent though fuzzy ideas were very much part of the mathematical apparatus needed to get the Newtonian revolution finally moving after two centuries of somnolence.

## Discovery of the Kalman Filter

In America in the early 1950s, the situation concerning Newton's discovery and invention, probably neither at that time as yet a recognizable revolution, was roughly like this.

At leading centers of applied learning, such as MIT, there was heavy emphasis on electric circuits and some also on simulation, modeling, control, communications, and signal theory. Newton and differential equations were way, way out of fashion. But Heaviside was in, thanks to having been repackaged and made respectable as the Laplace transform. The concept and even the definition of a linear system was unknown. Instead, resistance–inductance–capacitance (RLC) circuits and the like were used as prototype examples of finite, lumped-parameter systems, which were called linear if they satisfied the superposition principle. (They were expected to do that.) The sloppy notion of superposition was borrowed from the language (but not the mathematics or physics) of quantum mechanics, and it seemed as if no one had ever thought that linear was a mathematical concept.

Nonetheless, if somewhat ironically, mathematics enjoyed enormous prestige. Prestige in the deterrent sense. When Norbert Wiener (1894–1964) formulated filtering as a mathematical problem, the "yellow peril" (1943), it became a kind of Mount Everest, or, to be patriotic, a Mount McKinley, a symbol of what could be fashioned by mathematics using fancy gadgets like the Laplace transform. But there were also nasty underground rumors that Wiener filters were no good, hopelessly impractical, and too hard to automate.

As I have mentioned on numerous occasions (see Ref. 8, page 13), the discovery of the Kalman filter (January 1959) came about through a single, gigantic, persistent mathematical exercise. It was not my well-earned reward after long years of relentless research. (The acknowledgement of partial support in my first publication[9] referred to an umbrella grant to a large, heterogeneous group; this grant was not specifically related to the filtering problem.) Just as Newton was lucky having timed his birth so as to have Kepler's laws ready and waiting for him, I was lucky, too. The pieces of the slumbering Newtonian revolution, which I needed for my monster exercise, were available, scattered all around, partially forgotten but ready to be picked up and reassembled again.

Wiener was a mathematician of vast intellectual culture, but he had a superficial understanding of mathematics as living and growing knowledge. As it turned out, this was also my luck. Wiener filtering (initially called "extrapolation and interpolation of time series") was formulated with impeccable mathematical precision, with some perfunctory asides in the direction of the calculus of variations but without any feeling for, any attention, even lip service, to the "big picture." I, too, was blissfully innocent of seeing the big picture. (This always takes some time.) But I was not afraid to think mathematically. Soon I noticed serious flaws in Wiener's formulation. Two of these flaws were the following:

1) He took it for granted that statistical gadgets (like the time-correlation function) are the right way to encode quantitative uncertainty.

2) He was under the impression that a system description (like the transfer function) is exactly the same as a system in a concrete physical sense.

About the first flaw: This may have been the claim, or hope, of large-sample statistics in the early 1920s, but that hope was no longer credible 20 years later. It was absurd to think that data from aggregate statistical procedures are necessarily preferable to data from physical measurements, and equally absurd to imagine that direct processing, say, averaging, of physical data would be enough to get rid of noise. Nonetheless, a whole subculture, especially at MIT, arose around playing with, computing with, and examining on correlation functions. (See Ref. 10 and even the pioneering textbook/monograph Ref. 11.) Thus, if Wiener organized his main contribution to filtering around correlation functions, he was certainly not out of the MIT mainstream. But that didn't impress me. It seemed mathematically unnatural.

The way out was easy. I took a rigidly pure-mathematical point of view. In Ref. 9, I simply defined a stochastic signal source consisting of a linear system and discrete white noise, thereby "postponing" the thorny problem of how to bring in real data to validate such an abstract model. As I realized fairly soon, the thing that I have blithely postponed touched upon the core of the Newtonian revolution, except that classically the problem was dealt with only in the exact case of noise-free data (exact = noise free = nonstochastic = deterministic). My signal source was almost deterministic, a concrete system with its physical parameters given a priori; stochastics entered into the picture only as the white noise in the environment of the system.

Wiener may have believed that statistics has already solved this problem, simply by inventing correlation functions. But correlation functions do not fit nicely into the Newtonian scheme. It is very hard to carry out the step: stochastic data → model if the stochastic data is a correlation function. Indeed, the correlation function may turn out to be the wrong kind of condensation of data when our main interest is finding mathematically natural models. This problem is still poorly understood 50 years later.

About the second flaw: Now we are at the heart of the Newtonian revolution. We are asked to replace the behavioral description (transfer function or correlation function) of a system by a mathematically equivalent real thing, a physical, explicit, realizable system. This kind of thinking was foreign to Wiener, but it was natural and easy for me. My inspiration came in part from the investigation of nonlinear systems by Nicholas Minorsky (1885–1970), Russian–American engineer/mathematician, and in part from one of my later mentors, Solomon Lefschetz (1884–1972), Moskva-born, French-educated, American mathematician, who, after the age of 70, was concentrating all his energies on reviving mathematical research on nonlinear ordinary differential equations. Starting as an MIT undergraduate, for some 10 years I had been working hard to establish for myself first the obvious relations and then the precise equivalence between transfer functions and linear vector differential equations. By the early 1960s, this resulted in a nice generalization of Newton's theorem:

> Linear systems described by a transfer function matrix
> $\Leftrightarrow$
> linear vector differential equations (which are completely controllable and observable).

This new theorem, although not yet in its final form when researching Ref. 9, was very helpful in guiding me through the huge though quite straightforward exercise of redoing Wiener the right way, at the end of which was the discovery of the Kalman filter.

And, yes, this was a true discovery because of the following:

1) No one imagined that the end result would be That simple. (In a tutorial monograph,[10] organized around correlation functions and written by a student of Wiener, it took more than 400 pages to solve the simplest case that, using the new method, took me but a few lines.)

2) No one expected the result to be That general (no restrictions like infinite time interval, constant coefficients, etc.).

3) Everyone was happily surprised that it turned out to be That useful.

## Reflections

Of course, I was well aware how very, very important this discovery was. I even tried to explain it to girlfriends. But, honestly, I didn't quite imagine that it will turn out to be That important.

I have still not yet heard of the Newtonian revolution. I did, however, reflect on the course of events preceding and following Ref. 9. The biggest surprise to me was that my early claim to fame actually derived from a simple tactical decision I have taken around 1954 as a disgruntled graduate student. This was to attempt to

> develop linear system theory along the lines of linear algebra.

The program was easy and doable, therefore highly interesting. Good books in linear algebra and matrix theory were available as were good and talented friends such as Robert W. Bass (1930– ). But, to repeat, not only the concept of a system but even the definition of linear—and this definition is inconceivable without bringing in linear algebra—was murky in the engineering world of the 1950s. It is still murky today in some prestigious fields.

In any case and by any degree of hindsight, to try to model linear system theory on linear algebra was an excellent and inspired decision. It was to lead rapidly to the "realgebraization" of the systems field that was getting badly messed up by propaganda from the side of the Laplace transform. [Admittedly, the Laplace transform was pedagogically better suited for teaching linear systems by people who did not understand systems to people who will never understand systems—if I may be forgiven for paraphrasing here a remark by Arnol'd (Ref. 7, page 48) not unrelated to Newton.] My program exerted influence well beyond my hopes and my own contributions, and it helped to inject new vigor into both the offspring (linear systems) and the parent (linear algebra[12]).

Return to Ref. 9. Upon its public presentation (1 April 1959, Cleveland), the Kalman filter was at once praised (Session Chairman: too many summation signs for me but surely it must be something very important), copied (USSR Academy: please send us 20 reprints), applied whenever possible, and very little criticized. However, the underlying message about the Newtonian revolution, that the essential intellectual contribution in Ref. 9 came from mathematics, not physics or engineering, refused to sink in for a very long time.

For example, in a retrospective paper[13] written nearly 25 years later, the author (whose doctorate in applied mathematics predates mine in engineering by 6 years) noted (Ref. 13, page 106) that my "classic paper"[9] was too "esoteric" and highly "abstruse" compared to his own work, and then, oddly (Ref. 13, page 110), he forgot to list his own references 19 and 20 (= Ref. 9 here) central to these astute psychological observations. Unlike Feynman's complaint, that Newton's "arcane" proof (conic sections, 2000-year-old Greek mathematics) was too hard now to understand, the complaints in Ref. 13 about my mathematics in Ref. 9 were directed against modern linear algebra and probability theory, which I presented in the simplest possible way because that is how I learned to do these things myself a short time before. It would seem reasonable to conclude, therefore, that the real culprit (or hero) here must be mathematics, not one of its ancient or modern incarnations.

From the time when the *Principia* was in the process of publication, there is extant a famous letter by Newton (see Ref. 7, page 26) lamenting that he was cruelly underappreciated as a mathematician that he insisted he was. Had I felt underappreciated as a mathematician, a similar case could have been made in regard to the Kalman filter. But I didn't feel underappreciated.

Let us try to forget, for a moment, about mathematics or philosophizing about the know-why. More usefully, let us note that the know-how represented by the Kalman filter soon penetrated flight and space guidance and then the other new technologies already mentioned, and indeed all other varieties of system problems where noise and uncertainty were perceived as not just a nuisance but as a performance limitation, as in the global positioning system. Beyond these, there are many, many examples of interaction of the Kalman filter with softer fields like statistics, theoretical and applied economics, finance, etc.

## The Kalman Filter as an Invention

And is the Kalman filter really that useful? How can that be explained? Indeed here, too, there is a direct parallel with Newton's invention: Once the data are mathematized, you can do everything else by mathematics. In most applications, the actual Kalman filter is not a physical part of a machine like the fuel filter is of a gasoline engine. The material realization of the filter is generally a computer program. Once the signal source has been represented in exact mathematical form, all further analysis, optimization, control strategies, signal processing, etc., are carried out in the mathematical domain; this process ends in a computer program. So the invention of Newton has metamorphosed into a modern insight:

It can be done in software.

Among my many contemporaries who would have had this insight, my impression is that it is Bill Gates who deserves a lion's share of credit. Of course, one needs a computer. It wasn't Newton's fault that he (and we) had to wait 300 years before his invention became practical, and it was Gates's merit that we didn't have to wait too much longer after the computer was born.

As regards the invention of the Kalman filter, the inventor and the invention, both, should consider themselves lucky to have been around just at the time when, after long and anxious gestation, real-time computing was preparing to take off for the stars.[14] (If you prefer to get back to Earth, please note that not everything can be done in software, for example, powered, controlled, manned flight.)

## The Darker Side

All of these issues are inevitable, obvious, clear, and easy. But are they really? Three hundred years after the *Principia* the miracle of the Newtonian revolution, and, within it, the Kalman filter, has been seen to work in innumerable successful applications. No need for further selling. But do we know when it works? How it works? Why it works?

As an example, the Kalman filter does not seem to work in the stock market. I receive neither thanks nor money.

In our times, one who gave much thought to the miracle aspect of the Newtonian revolution was Eugene P. Wigner (1902–1995), the great Hungarian/American theoretical physicist. In a marvelous paper,[15] the title of which is already an abstract and almost the whole story, he looked at planetary (Keplerian) motion, through the physicist's eyes, and wondered. How could Newton have been so confident in his gravitational law (later verified to better than one part in a million) when his initial data were accurate only to some 4%?

This was just a few years before Feynman's lost lecture I mentioned earlier. Both committed the same psychological misstep: looking at Newton's great discovery with physical instead of mathematical eyes. We have seen that Newton's deductions were mathematical and his data was exact. Because Kepler's laws are exact. But the data used by Kepler, while very good, was not exact. Thereby, the burden of explaining the miracle shifts from Newton onto Kepler. This is simply the question of science. Mathematics cannot generate basic scientific information about the real world (how big is the Earth? why is the grass green?); at most it can digest data given to it.

This issue is particularly acute for the Kalman filter. Data needed to construct it is based, in part, on "random processes" (see the title of Ref. 11). This is not part of Keplerian or Newtonian science. A random process is an abstract mental construct, not something that can be pieced together from measurements. Procedures for determining parameters for a random process are indirect. If a random process exists in the axiomatic sense used in Ref. 9—and there are some, including the present writer, who may deny this—then all is well with the Kalman filter because it is all mathematics. If not, the question is very much open. The burden rests on those who want to apply the Kalman filter at all costs, even when they have lousy data, or when they don't understand the relevant science, or when there is simply no relevant science. (The stock market has all of these.)

If we take a rigidly Newtonian position, banning all hypotheses, the Kalman filtering problem must be formulated as follows: What do you do to process finite, real-time data, in a noisy environment, so that the output of your computer program constitutes a Kalman filter? No hypotheses about signal generation, probability, white noise, randomness are allowed; these notions, if they exist in the real world and are relevant to the filtering problem, should pop out of the exercise, just as the gravitational law came out of Newton's exercise "hypotheses non fingo," without resorting to a hypothesis. The success of this and similar programs depends upon whether the Newtonian revolution will keep right on going. My private guess is that it will.

The present paper is a summary of what I have learned over many years. For an earlier outline of my position, please look at Ref. 16.

## Acknowledgment

This work is a revised version of a public lecture at the University of Athens, 22 May 2003.

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